

Radiatively Generated Parton Distributions of Polarized Hadrons and Photons*

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Abstract

A next-to-leading order QCD analysis of spin asymmetries and structure functions in polarized deep inelastic lepton nucleon scattering is presented within the framework of the radiative parton model. The Q^2 -dependence of the spin asymmetry $A_1^p(x, Q^2)$ is shown to be non-negligible for x -values relevant for the analysis of the present data. In the second part of the paper we present LO results for radiatively generated spin-dependent parton distributions of the photon and for its structure function g_1^γ .

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1 Introduction

The past few years have seen much progress in our knowledge about the nucleons' spin structure due to the experimental study of the spin asymmetries $A_1^N(x, Q^2) \approx g_1^N(x, Q^2)/F_1^N(x, Q^2)$ ($N = p, n, d$) in deep-inelastic scattering (DIS) with longitudinally polarized lepton beams and nucleon targets. Previous data on A_1^p by the SLAC-Yale collaboration [1] have been succeeded by more accurate data from [2-4], which also cover a wider range in (x, Q^2) , and results on A_1^n and A_1^d have been published in [5] and [6, 7], respectively.

On the theoretical side, it has become possible to perform a complete and consistent study of polarized DIS in next-to-leading order (NLO) of QCD, since the calculation of the spin-dependent two-loop anomalous dimensions, needed for the NLO evolution of polarized parton distributions, has been completed recently [8]. A first such study has been presented in [9], where the underlying concept has been the radiative generation of parton distributions from a low resolution scale μ , which in the unpolarized case had previously led [10] to the remarkably successful prediction of the small- x rise of the proton structure function F_2^p as observed at HERA [11]. The main findings of this NLO analysis [9], which followed the lines of an earlier leading order (LO) study [12], will be collected in section 2.

New precise data on polarized DIS will be added in the near future from the HERMES experiment [13] at HERA. Moreover, it is no longer inconceivable to longitudinally polarize HERA's high-energy proton beam. The corresponding situation with unpolarized beams has already demonstrated [11] that at such high energies the ep interactions, since dominated by the exchange of almost real (Weizsäcker-Williams) photons, can reveal also information on the parton content of the *photon* in addition to that of the proton. Therefore, a polarized $\vec{e}\vec{p}$ -collider mode of HERA could in principle serve to explore the spin-dependent parton distributions of circularly polarized photons. These are completely unmeasured and thus unknown up to now, and one has to invoke models [14] in order to study their expected size and to estimate the theoretical uncertainty in predictions for them. For this purpose, it seems worthwhile to also resort to a radiative generation of the *photon's* polarized parton distributions [14], since the corresponding predictions for the unpolarized photon [15] have again been phenomenologically successful [11]. This topic will be covered in section 3.

2 NLO Radiative Parton Model Analysis of Polarized DIS

Measurements of polarized deep inelastic lepton nucleon scattering yield direct information [1-7] on the spin-asymmetry

$$A_1^N(x, Q^2) \simeq \frac{g_1^N(x, Q^2)}{F_1^N(x, Q^2)} = \frac{g_1^N(x, Q^2)}{F_2^N(x, Q^2)/[2x(1 + R^N(x, Q^2))]} \quad , \quad (1)$$

where $N = p, n, d$ and $R \equiv F_L/2xF_1 = (F_2 - 2xF_1)/2xF_1$. In NLO, $g_1^N(x, Q^2)$ is related to the polarized (Δf^N) quark and gluon distributions in the following way:

$$\begin{aligned} g_1^N(x, Q^2) = & \frac{1}{2} \sum_q e_q^2 \left\{ \Delta q^N(x, Q^2) + \Delta \bar{q}^N(x, Q^2) + \right. \\ & \left. + \frac{\alpha_s(Q^2)}{2\pi} [\Delta C_q * (\Delta q^N + \Delta \bar{q}^N) + \Delta C_g * \Delta g] \right\} \end{aligned} \quad (2)$$

with the convolutions (*) being defined as usual, and where the appropriate spin-dependent Wilson coefficients ΔC_i in the $\overline{\text{MS}}$ scheme are given, e.g., in [8]. The NLO form of the unpolarized (spin-averaged) structure function $F_1^N(x, Q^2)$ is similar to the one in (2) with $\Delta f^N(x, Q^2) \rightarrow f^N(x, Q^2)$ and the unpolarized Wilson coefficients given, for example, in [16].

The NLO Q^2 -evolution of the spin-dependent parton distributions $\Delta f(x, Q^2) \equiv \Delta f^p(x, Q^2)$ is performed most conveniently in Mellin n -moment space where the solutions of the evolution equations (see, e.g., refs.[16-18]) can be obtained analytically, once the boundary conditions at some $Q^2 = \mu^2$, i.e. the input densities $\Delta f(x, \mu^2)$ to be discussed below, are specified. These Q^2 -evolutions are governed by the spin-dependent LO [19] and NLO [8] ($\overline{\text{MS}}$) anomalous dimensions. Having obtained the analytic NLO solutions for the moments of parton densities it is simple to (numerically) Mellin-invert them to Björken- x space as described, for example, in [17, 18]. As seen in (2), the so obtained $\Delta f(x, Q^2)$ are then convoluted with the Wilson coefficients ΔC_i to yield the desired $g_1(x, Q^2)$.

To fix the polarized NLO input parton distributions $\Delta f(x, Q^2 = \mu^2)$ we perform fits to the directly measured asymmetry $A_1^N(x, Q^2)$ in (1), rather than to the derived $g_1^N(x, Q^2)$, mainly because for the experimental extraction of the latter often the assumption of the Q^2 -independence of $A_1^N(x, Q^2)$ is made, which is theoretically not justified as we will see below. As mentioned in the introduction, the other main ingredient of our NLO analysis [9] is that we follow the radiative (dynamical) concept [10] by choosing the low input scale $Q^2 = \mu^2 = 0.34 \text{ GeV}^2$ and implementing the fundamental positivity constraints

$$|\Delta f(x, Q^2)| \leq f(x, Q^2) \quad (3)$$

down to $Q^2 = \mu^2$. Therefore we shall use all presently available data [2-7] in the small- x region where $Q^2 \gtrsim 1 \text{ GeV}^2$, without introducing lower cuts in Q^2 as was usually necessary in

previous analyses [20]. A further advantage of this approach is the possibility to study the Q^2 -dependence of $A_1^N(x, Q^2)$ over a wide range of Q^2 which might be also relevant for possibly forthcoming polarization experiments at HERA. The analysis affords some well established set of unpolarized NLO parton distributions $f(x, Q^2)$ for calculating $F_1^N(x, Q^2)$ in (1) which will be adopted from ref.[10].

In addition to (3), the polarized NLO parton distributions $\Delta f(x, Q^2)$ are, for the $SU(3)_f$ symmetric 'standard' scenario, constrained by the sum rules

$$\int_0^1 dx \left(\Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d} \right) (x, \mu^2) = g_A = F + D = 1.2573 \pm 0.0028 \quad (4)$$

$$\int_0^1 dx \left(\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2(\Delta s + \Delta \bar{s}) \right) (x, \mu^2) = 3F - D = 0.579 \pm 0.025 \quad (5)$$

with the values of g_A and $3F - D$ taken from [21]. It should be noted that the first moments of the flavor non-singlet combinations which appear in (4) and (5) are Q^2 -independent also in NLO [22, 9, 23].

As a plausible alternative to the full $SU(3)_f$ symmetry between charged weak and neutral axial currents required for deriving the 'standard' constraints (4) and (5), we consider a 'valence' scenario [12, 24] where this flavor symmetry is broken and which is based on the assumption [24] that the flavor changing hyperon β -decay data fix only the total helicity of *valence* quarks:

$$\int_0^1 dx \left(\Delta u_v - \Delta d_v \right) (x, \mu^2) = g_A = F + D = 1.2573 \pm 0.0028 \quad (4')$$

$$\int_0^1 dx \left(\Delta u_v + \Delta d_v \right) (x, \mu^2) = 3F - D = 0.579 \pm 0.025 \quad (5')$$

We note that in both above scenarios the Björken sum rule manifestly holds due to the constraints (4),(4'). Our optimal NLO input distributions at $Q^2 = \mu^2 = 0.34 \text{ GeV}^2$ subject to these constraints can be found in [9].

A comparison of our results with the data on $A_1^N(x, Q^2)$ is presented in Fig.1. Obviously, very similar results are obtained for the two scenarios considered above. The Q^2 -dependence of $A_1^p(x, Q^2)$ is presented in Fig.2 for some typical fixed x values for $1 \leq Q^2 \leq 20 \text{ GeV}^2$ relevant for present experiments. In the (x, Q^2) region of present data [2-7], $A_1^p(x, Q^2)$ increases with Q^2 for $x > 0.01$. Therefore, since most present data in the small- x region correspond to small values of $Q^2 \gtrsim 1 \text{ GeV}^2$, the determination of $g_1^p(x, Q^2)$ at a larger fixed Q^2 (5 or 10 GeV^2 , say) by assuming $A_1^p(x, Q^2)$ to be independent of Q^2 , as is commonly done [2-7], is misleading and might lead to an *underestimate* of g_1^p by as much as about 20%. Results for the (also non-negligible) Q^2 -dependence of A_1^n can be found in [9, 12]. The assumption of approximate scaling for $A_1(x, Q^2)$ is therefore unwarranted and, in any case, theoretically not justified [9, 12, 25, 26].

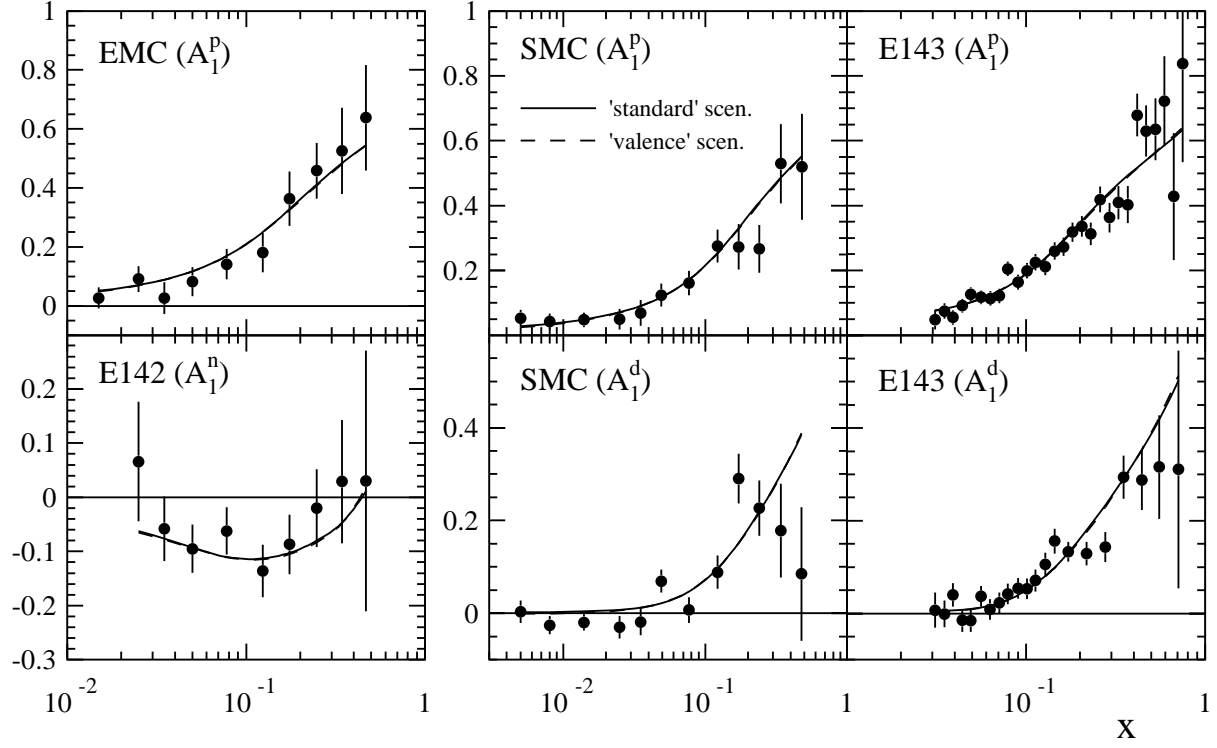


Figure 1: Comparison of our NLO results for $A_1^N(x, Q^2)$ as obtained from the fitted inputs at $Q^2 = \mu^2$ for the 'standard' and 'valence' scenarios with present data [2-7]. The Q^2 values adopted here correspond to the different values quoted in [2-7] for each data point.

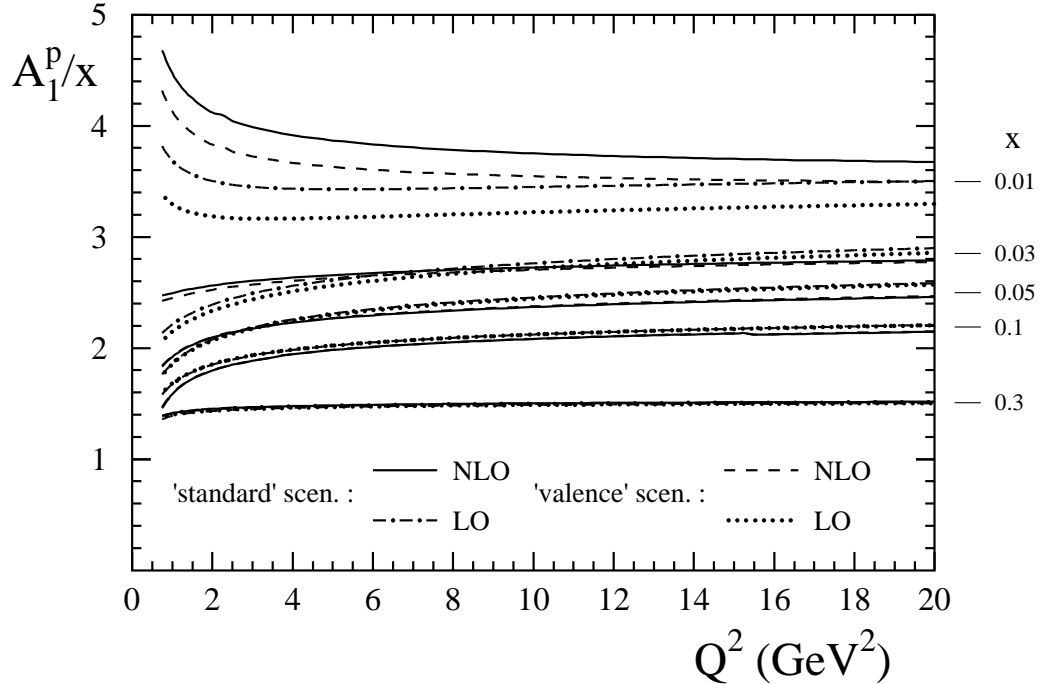


Figure 2: The Q^2 -dependence of $A_1^p(x, Q^2)$ as predicted by the NLO QCD evolution at various fixed values of x . The LO results are from [12].

A further result of our analysis is that the polarized gluon density $\Delta g(x, Q^2)$ is hardly constrained by present experiments. Similarly agreeable fits as those shown in Fig.1 to all present asymmetry data can also be obtained for a fully saturated gluon input $\Delta g(x, \mu^2) = g(x, \mu^2)$ as well as for the less saturated $\Delta g(x, \mu^2) = xg(x, \mu^2)$ or even a purely dynamical input $\Delta g(x, \mu^2) = 0$. We compare such gluons at $Q^2 = 4 \text{ GeV}^2$ in Fig.3. The variation of $\Delta g(x, Q^2)$ allowed by present experiments is indeed sizeable. This implies, in particular, that the Q^2 -evolution of $g_1(x, Q^2)$ below the experimentally accessible x -range is not predictable for the time being.

To conclude this section, our results [9, 12] demonstrate the compatibility of the restrictive radiative model [10] with present measurements of deep inelastic spin asymmetries and structure functions.

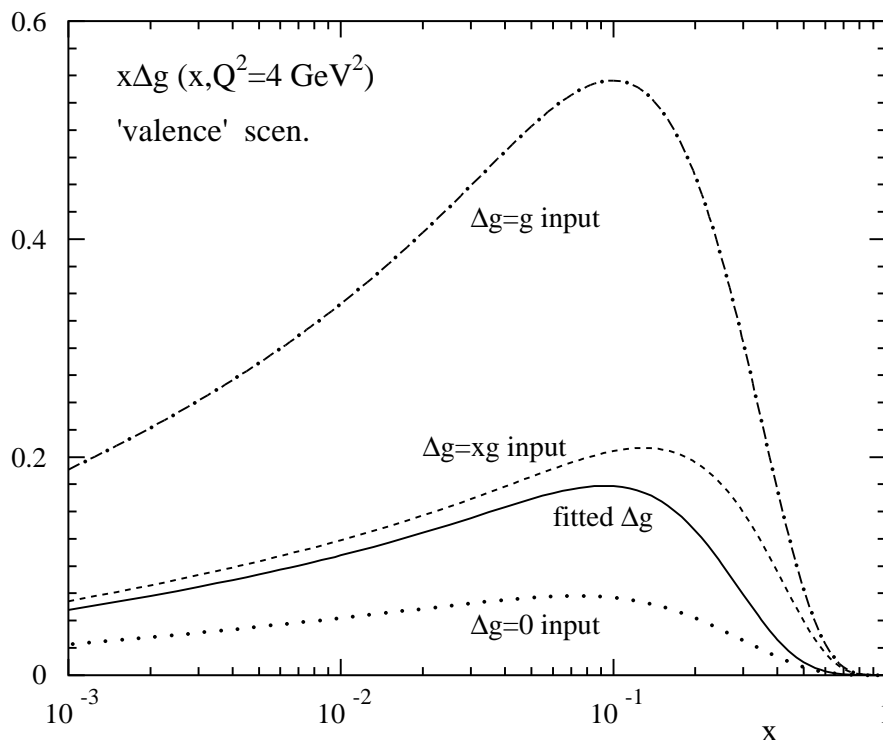


Figure 3: The experimentally allowed range of polarized gluon densities at $Q^2 = 4 \text{ GeV}^2$ for the 'valence' scenario with differently chosen $\Delta g(x, \mu^2)$ inputs.

3 Spin-dependent parton distributions of the photon

As mentioned in the introduction, the photon's polarized parton distributions, $\Delta f^\gamma = \Delta q^\gamma$, Δg^γ , are presently unmeasured and thus completely unknown. Theoretical expectations can, however, be derived [14] by assuming a radiative generation of the Δf^γ along the lines followed [15] successfully for the unpolarized photonic parton distributions, $f^\gamma = q^\gamma, g^\gamma$. In [15] a VMD valence-like structure at the low resolution scale $Q^2 = \mu^2 = 0.25 \text{ GeV}^2$ was imposed as the

input boundary condition, assuming that at this resolution scale the photon behaves like a vector meson, i.e., that its parton content is proportional to that of the ρ -meson. Since nothing is known experimentally about the latter, the parton densities of the neutral pion as determined in a previous study [27] were used instead which are expected not to be too dissimilar from those of the ρ . Unfortunately, this procedure is obviously impossible for determining the VMD input distributions $\Delta f^\gamma(x, \mu^2)$ for the *polarized* photon. Some help is again provided by the positivity constraints

$$|\Delta f^\gamma(x, \mu^2)| \leq f^\gamma(x, \mu^2) \quad , \quad (6)$$

and it is interesting to see how restrictive these general conditions already are. For this purpose we consider [14] two very different scenarios with 'maximal', $\Delta f^\gamma(x, \mu^2) = f^\gamma(x, \mu^2)$, and 'minimal', $\Delta f^\gamma(x, \mu^2) = 0$, saturation of (6). We mention that a sum rule expressing the vanishing of the first moment of the polarized photon structure function g_1^γ was derived from current conservation in [28] which, in the LO considered here, is equivalent to

$$\int_0^1 \Delta q^\gamma(x, \mu^2) = 0 \quad . \quad (7)$$

This sum rule can in principle serve to further restrict the range of allowed VMD inputs. On the other hand, we are interested only in the region of, say, $x > 0.01$ here, such that the current conservation constraints (7) could be implemented by contributions from smaller x . To estimate the uncertainties in the predictions for the $\Delta f^\gamma(x, Q^2)$ stemming from the insufficiently known VMD input we therefore stick to the two extreme scenarios discussed above, even though strictly speaking the maximally saturated input violates the sum rule (7).

Starting from the two different boundary conditions for $\Delta f^\gamma(x, \mu^2)$ we generate $\Delta f^\gamma(x, Q^2)$ at $Q^2 > \mu^2$ as in [15] replacing the unpolarized splitting functions in the Q^2 -evolution equations by their polarized counterparts [19]. In view of the uncertainties in the input, we restrict our calculations to the leading order, although in principle a NLO analysis has become possible now [29] by exploiting the results for the polarized two-loop splitting functions of ref. [8]. The resulting parton asymmetries [14]

$$A_f^\gamma(x, Q^2) \equiv \frac{\Delta f^\gamma(x, Q^2)}{f^\gamma(x, Q^2)} \quad (8)$$

are shown in Fig.4 for $Q^2 = 30 \text{ GeV}^2$. As can be seen, there are quite substantial differences between the results for the two scenarios. Fig.5 shows the corresponding LO results for the polarized photon structure function

$$g_1^\gamma(x, Q^2) \equiv \sum_q e_q^2 \Delta q^\gamma(x, Q^2) \quad , \quad (9)$$

where charm contributions from the subprocesses $\gamma^* \gamma \rightarrow c\bar{c}$ and $\gamma^* g \rightarrow c\bar{c}$ have been included (see [30] for further details). For comparison we also include in Fig.5 the 'asymptotic' result

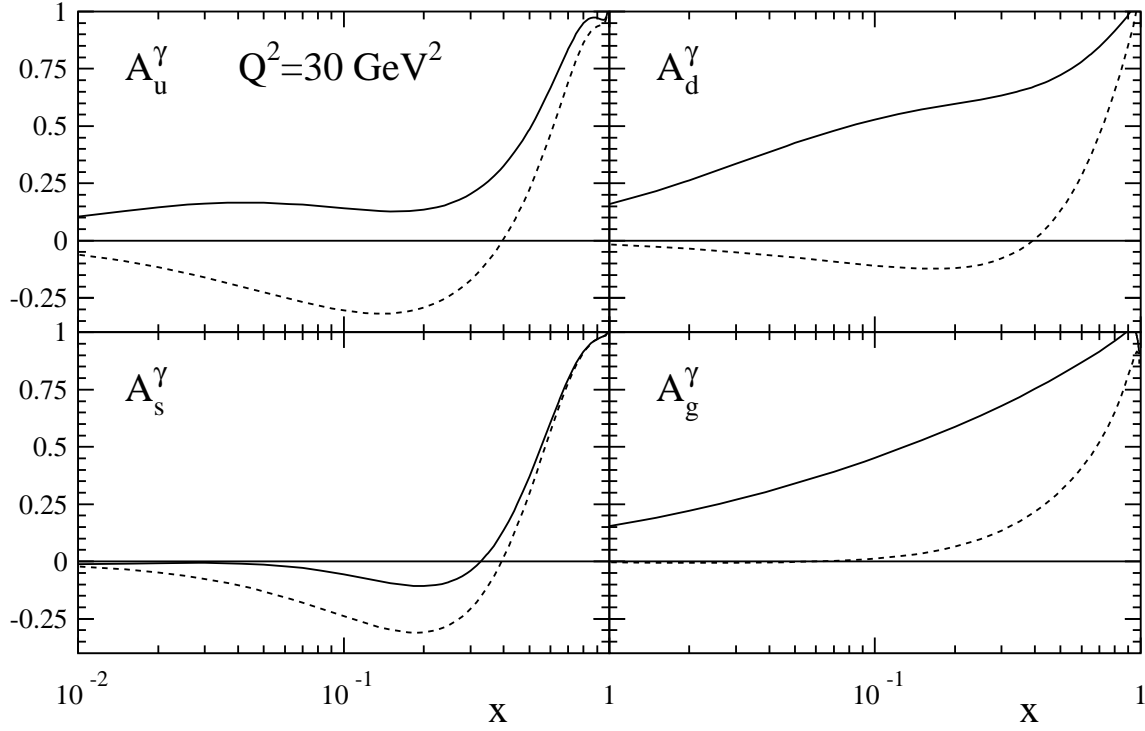


Figure 4: The predicted photonic parton asymmetries $A_f^\gamma(x, Q^2)$ at $Q^2 = 30 \text{ GeV}^2$ as defined in (8) for 'maximal' (solid lines) and 'minimal' (dashed lines) saturation of the VMD input.

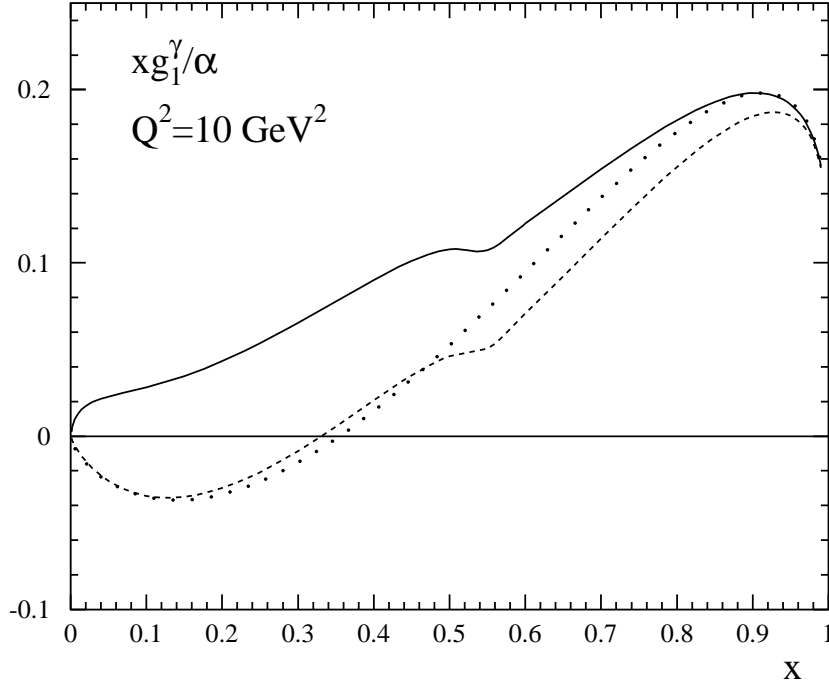


Figure 5: LO predictions for the photon's spin-dependent structure function $xg_1^\gamma(x, Q^2)/\alpha$ at $Q^2 = 10 \text{ GeV}^2$ for 'maximal' (solid) and 'minimal' (dashed) saturation of the VMD input. The dotted line shows the result for the LO 'asymptotic' solution for 3 active flavors also considered in [31].

for g_1^γ which was also considered in refs. [31]. g_1^γ would in principle be accessible in polarized e^+e^- collisions [30] or in $\vec{e}\vec{\gamma}$ processes. In a polarized collider mode of HERA, the spin-dependent parton distributions of the photon will show up in $\vec{\gamma}\vec{p}$ processes like jet [32] or heavy flavor photoproduction when the photon is resolved into its hadronic structure [29].

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